Analytic Geometry Giant Review Answers

1. $m = \frac{3}{4}$

$P(-2, -2)$

$y = mx + b$

$(-2) = \left(\frac{3}{4}\right)\left(\frac{-2}{1}\right) + b$

$-2 = \frac{-3}{2} + b$

$-\frac{2 \times 2}{1 \times 2} + \frac{3}{2} = b$

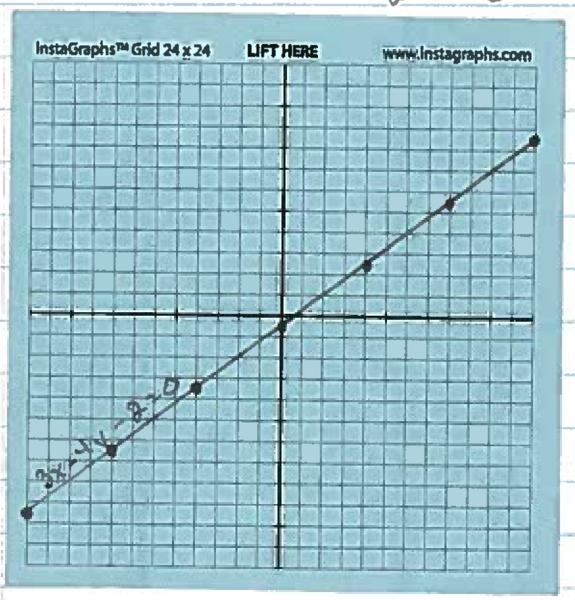
$-\frac{4}{2} + \frac{3}{2} = b$

$-\frac{1}{2} = b$

$\therefore y = \frac{3}{4}x - \frac{1}{2}$

$4\left(\frac{3}{4}x\right) - 4(y) - 4\left(\frac{1}{2}\right) = 4(0)$

$\boxed{3x - 4y - 2 = 0}$



2. $(2, 7)$

$(-3, 9)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{9 - 7}{-3 - 2}$

$= \frac{2}{-5}$

$y = mx + b$

$y = \left(-\frac{2}{5}\right)x + b$

$7 = \left(-\frac{2}{5}\right)\left(\frac{2}{1}\right) + b$

$7 = \frac{-4}{5} + b$

$\frac{7 \times 5}{1 \times 5} + \frac{4}{5} = b$

$\frac{35}{5} + \frac{4}{5} = b$

$\frac{39}{5} = b$

$\therefore y = -\frac{2}{5}x + \frac{39}{5}$

$5\left(\frac{2}{5}x\right) + 5(y) - 5\left(\frac{39}{5}\right) = 5(0)$

$\boxed{2x + 5y - 39 = 0}$

3. $y = -3x - 18 \rightarrow \perp m = \frac{1}{3}$

$P(-2, 3)$

$y = mx + b$

$y = \left(\frac{1}{3}\right)x + b$

$3 = \left(\frac{1}{3}\right)\left(\frac{-2}{1}\right) + b$

$3 = \frac{-2}{3} + b$

$\frac{3 \times 3}{1 \times 3} + \frac{2}{3} = b$

$\frac{9}{3} + \frac{2}{3} = b$

$\frac{11}{3} = b$

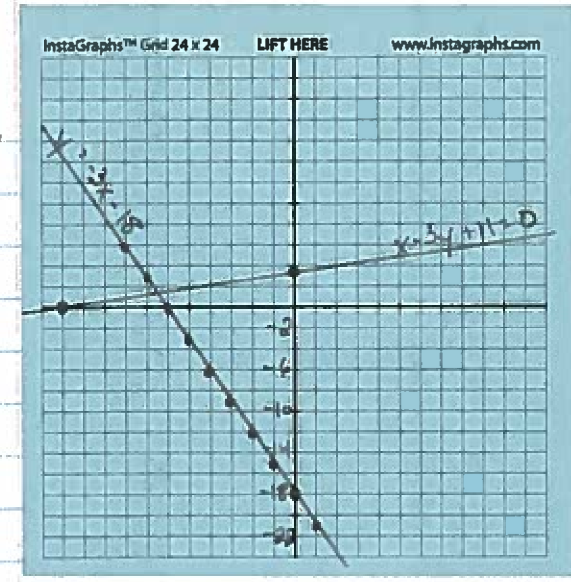
$\therefore y = \frac{1}{3}x + \frac{11}{3}$

$3\left(\frac{1}{3}x\right) - 3(y) + 3\left(\frac{11}{3}\right) = 3(0)$

$\boxed{x - 3y + 11 = 0}$

*Graphs on back

* 3.



$y = -3x - 18 \rightarrow m = -3,\ b = -18$

$x - 3y + 11 = 0$

$(0) - 3y = -11$

$-3y = -11$

$\frac{-3y}{-3} = \frac{-11}{-3}$

$y = \frac{11}{3} \rightarrow 3.7$

$(0, 3.7)$

$x - 3(0) = -11$

$x - 0 = -11$

$x = -11$

$(-11, 0)$

4. $A(5, -3)$
$B(4, 4)$

$M_{AB} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{5+4}{2}, \frac{-3+4}{2}\right)$

$= \left(\frac{9}{2}, \frac{1}{2}\right)$

5. $A(7, 3)$
$B(4, -1)$

$d_{AB} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(4-7)^2 + (-1-3)^2}$

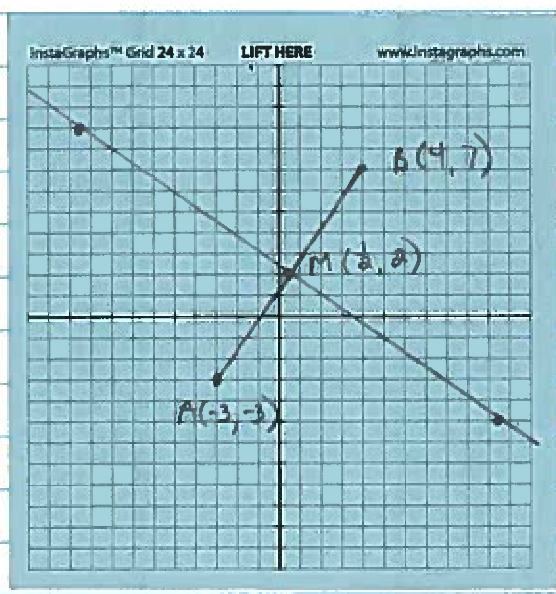
$= \sqrt{(-3)^2 + (-4)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= 5$ units

6. $A(-3, -3)$
$B(4, 7)$



① $M_{AB} = \left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$

$= \left(\frac{-3+4}{2}, \frac{-3+7}{2}\right)$

$= \left(\frac{1}{2}, \frac{4}{2}\right)$

$= \left(\frac{1}{2}, 2\right)$

② $m_{AB} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-3-7}{-3-4}$

$= \frac{-10}{-7}$

$= \frac{10}{7} \rightarrow \perp m = \frac{-7}{10}$

③ $y = mx + b$

$(2) = \left(\frac{-7}{10}\right)\left(\frac{1}{2}\right) + b$

$2 = \frac{-7}{20} + b$

$\frac{2 \times 20}{1 \times 20} + \frac{7}{20} = b$

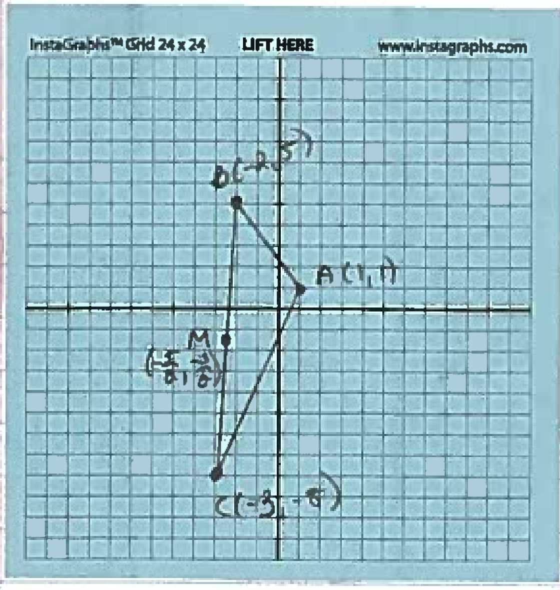
$\rightarrow \frac{40}{20} + \frac{7}{20} = b$

$\frac{47}{20} = b$

$\therefore y = \frac{-7}{10}x + \frac{47}{20}$

$20\left(\frac{7}{10}x\right) + 20(y) - 20\left(\frac{47}{20}\right) = 20(0) \rightarrow \boxed{14x + 20y - 47 = 0}$

7.



① $M_{BC} = \left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$

$= \left(\frac{-2+(-3)}{2}, \frac{5+(-8)}{2}\right)$

$= \left(\frac{-5}{2}, \frac{-3}{2}\right)$

② $m_{AM} = \dfrac{y_2-y_1}{x_2-x_1}$

$= \dfrac{-\frac{3}{2} - 1^{\times 2}_{\times 2}}{-\frac{5}{2} - 1^{\times 2}_{\times 2}}$

$= \dfrac{-\frac{3}{2} - \frac{2}{2}}{-\frac{5}{2} - \frac{2}{2}}$

$= \dfrac{-\frac{5}{2}}{-\frac{7}{2}} \rightarrow -\frac{5}{2} \div -\frac{7}{2}$

$= -\frac{5}{2} \times -\frac{2}{7}$

$= \frac{5}{7}$

③ $y = mx + b$

$y = \left(\frac{5}{7}\right)x + b$

$1 = \left(\frac{5}{7}\right)(1) + b$

$1 = \left(\frac{5}{7}\right) + b$

$\frac{1\times 7}{1\times 7} - \frac{5}{7} = b$

$\frac{7}{7} - \frac{5}{7} = b$

$\frac{2}{7} = b$

$\therefore y = \frac{5}{7}x + \frac{2}{7}$

$7\left(\frac{5}{7}x\right) - 7(y) + 7\left(\frac{2}{7}\right) = 7(0)$

$\boxed{5x - 7y + 2 = 0}$

$d_{AM_1} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{\left(-\frac{5}{2} - \frac{1\times 2}{1\times 2}\right)^2 + \left(-\frac{3}{2} - \frac{1\times 2}{1\times 2}\right)^2}$

$= \sqrt{\left(-\frac{5}{2} - \frac{2}{2}\right)^2 + \left(-\frac{3}{2} - \frac{2}{2}\right)^2}$

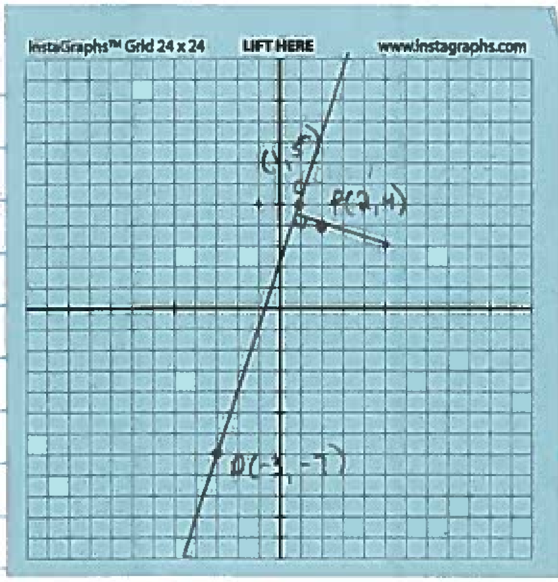
$= \sqrt{\left(-\frac{7}{2}\right)^2 + \left(-\frac{5}{2}\right)^2}$

$= \sqrt{\left(\frac{49}{4}\right) + \left(\frac{25}{4}\right)}$

$= \sqrt{\frac{74}{4}}$

$= \sqrt{\frac{37}{2}}$

8.



① $m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{5-(-7)}{1-(-3)}$

$= \frac{12}{4}$

$= 3 \rightarrow \perp m = -\frac{1}{3}$

② $y = mx + b$

$y = (-\frac{1}{3})x + b$

$4 = (-\frac{1}{3})(\frac{2}{1}) + b$

$4 = (-\frac{2}{3}) + b$

$\frac{4\times3}{1\times3} + \frac{2}{3} = b$

$\frac{12}{3} + \frac{2}{3} = b$

$\frac{14}{3} = b$

∴ $y = -\frac{1}{3}x + \frac{14}{3}$

$3(\frac{1}{3}x)\,3(+y)\,3(\frac{14}{3}) = 3(0)$

$\boxed{x + 3y - 14 = 0}$

*Need to find the eq'n of CD

$y = mx + b$

$y = 3x + b$

$5 = 3(1) + b$

$5 = 3 + b$

$5 - 3 = b$

$2 = b$

∴ $y = 3x + 2$

④ $y = 3x + 2$

$x + 3y - 14 = 0$

$x + 3(3x+2) - 14 = 0$

$x + 9x + 6 - 14 = 0$

$10x - 8 = 0$

$\frac{10x}{10} = \frac{8}{10}$

$x = \frac{4}{5}$

∴ Intersection point $(\frac{4}{5}, \frac{22}{5})$

$y = 3x + 2$

$y = \frac{3}{1}(\frac{4}{5}) + 2$

$y = \frac{12}{5} + \frac{2\times5}{1\times5}$

$y = \frac{12}{5} + \frac{10}{5}$

$y = \frac{22}{5}$

Verify

LS: $x + 3y - 14$

$= (\frac{4}{5}) + \frac{3}{1}(\frac{22}{5}) - 14$

$= \frac{4}{5} + \frac{66}{5} - 14$

$= \frac{70}{5} - 14$

$= 14 - 14$

$= 0$ ✓

RS $= 0$

$d_{PF} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(\frac{4}{5} - \frac{2\times5}{1\times5})^2 + (\frac{22}{5} - \frac{4\times5}{1\times5})^2}$

$= \sqrt{(\frac{4}{5} - \frac{10}{5})^2 + (\frac{22}{5} - \frac{20}{5})^2}$

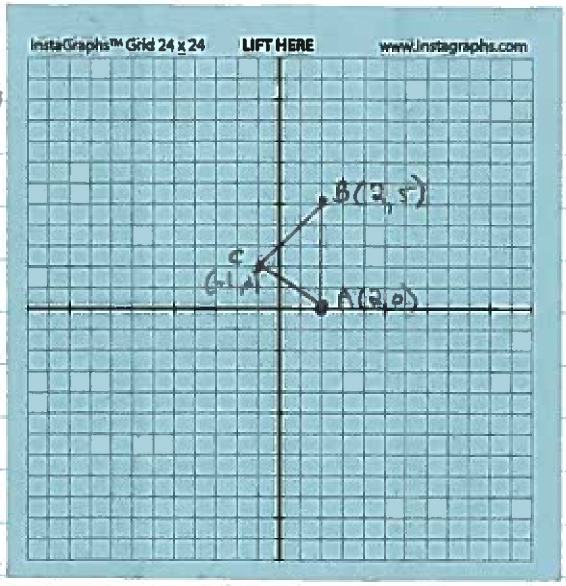
$= \sqrt{(-\frac{6}{5})^2 + (\frac{2}{5})^2}$

$= \sqrt{(\frac{36}{25}) + (\frac{4}{25})}$

$= \sqrt{\frac{40}{25}}$

$= \sqrt{\frac{8}{5}}$

9. a)



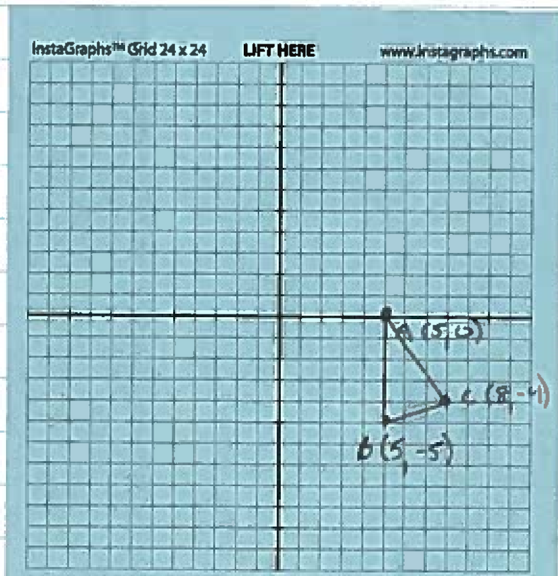
$$d_{AB} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(2-2)^2 + (5-0)^2}$$
$$= \sqrt{(0)^2 + (5)^2}$$
$$= \sqrt{0+25}$$
$$= \sqrt{25}$$
$$= 5$$

$$d_{BC} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(-1-2)^2 + (2-5)^2}$$
$$= \sqrt{(-3)^2 + (-3)^2}$$
$$= \sqrt{9+9}$$
$$= \sqrt{18}$$

∴ The lengths are all different, so △ABC is scalene.

$$d_{AC} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(-1-2)^2 + (2-0)^2}$$
$$= \sqrt{(-3)^2 + (2)^2}$$
$$= \sqrt{9+4}$$
$$= \sqrt{13}$$

b)



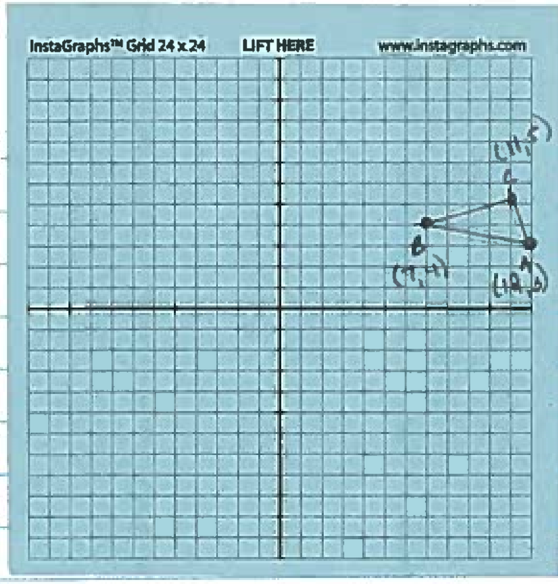
$$d_{AB} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(5-5)^2 + (-5-0)^2}$$
$$= \sqrt{(0)^2 + (-5)^2}$$
$$= \sqrt{0+25}$$
$$= \sqrt{25}$$
$$= 5$$

$$d_{BC} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(8-5)^2 + [-4-(-5)]^2}$$
$$= \sqrt{(3)^2 + (1)^2}$$
$$= \sqrt{9+1}$$
$$= \sqrt{10}$$

$$d_{AC} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(8-5)^2 + (-4-0)^2}$$
$$= \sqrt{(3)^2 + (-4)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$= 5$$

∴ AB = AC, so △ABC is isosceles.

9. c)



$d_{AB} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
$= \sqrt{(12-7)^2 + (3-4)^2}$
$= \sqrt{(5)^2 + (-1)^2}$
$= \sqrt{25 + 1}$
$= \sqrt{26}$

$d_{BC} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
$= \sqrt{(11-7)^2 + (5-4)^2}$
$= \sqrt{(4)^2 + (1)^2}$
$= \sqrt{16 + 1}$
$= \sqrt{17}$

$\therefore$ All lengths are different, so $\triangle ABC$ is scalene.

$d_{AC} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
$= \sqrt{(11-12)^2 + (5-3)^2}$
$= \sqrt{(-1)^2 + (2)^2}$
$= \sqrt{1 + 4}$
$= \sqrt{5}$

d)

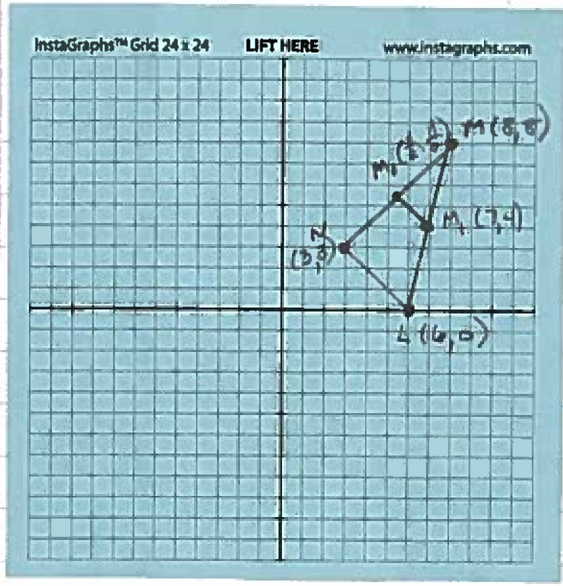
$d_{AB} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
$= \sqrt{(0-4)^2 + (4-5)^2}$
$= \sqrt{(-4)^2 + (-1)^2}$
$= \sqrt{16 + 1}$
$= \sqrt{17}$

$d_{BC} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
$= \sqrt{(0-1)^2 + (1-0)^2}$
$= \sqrt{(-1)^2 + (1)^2}$
$= \sqrt{16 + 1}$
$= \sqrt{17}$

$\therefore AB = BC$, so $\triangle ABC$ is isosceles.

$d_{AC} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
$= \sqrt{(1-4)^2 + (0-5)^2}$
$= \sqrt{(-3)^2 + (-5)^2}$
$= \sqrt{9 + 25}$
$= \sqrt{34}$

10.



$$m_{MN} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-8}{3-8} = \frac{-5}{-5} = 1$$

$$m_{NL} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3}{6-3} = \frac{-3}{3} = -1$$

$$m_{ML} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-8}{6-8} = \frac{-8}{-2} = 4$$

∴ Since the slopes of MN + NL are negative reciprocals, $\triangle LMN$ is a right triangle.

11. $M_{MN} = \left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right) = \left(\frac{8+3}{2}, \frac{8+3}{2}\right) = \left(\frac{11}{2}, \frac{11}{2}\right)$

$M_{NL} = \left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right) = \left(\frac{3+6}{2}, \frac{3+0}{2}\right) = \left(\frac{9}{2}, \frac{3}{2}\right)$

$M_{LM} = \left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right) = \left(\frac{8+6}{2}, \frac{8+0}{2}\right) = \left(\frac{14}{2}, \frac{8}{2}\right) = (7, 4)$

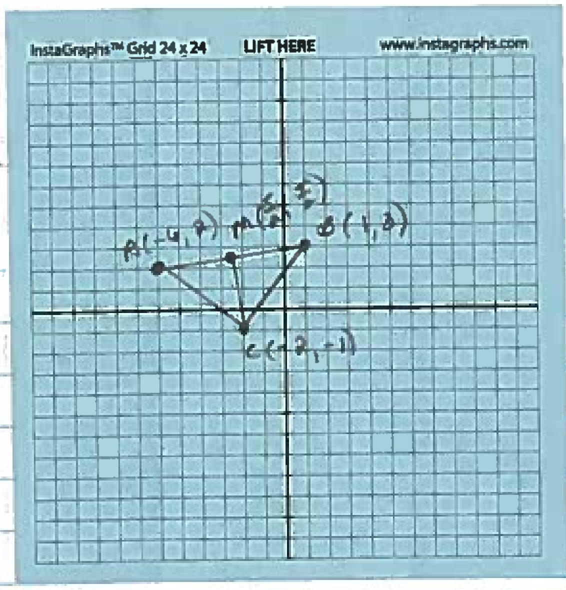
* Find the slope from the M_{MN} to the M_{ML}.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4^{\times 2}_{\times 2} - \frac{11}{2}}{7^{\times 2}_{\times 2} - \frac{11}{2}} = \frac{\frac{8}{2} - \frac{11}{2}}{\frac{14}{2} - \frac{11}{2}} = \frac{-\frac{3}{2}}{\frac{3}{2}} \rightarrow -\frac{3}{2} \div \frac{3}{2} = -\frac{\cancel{3}^{1}}{\cancel{2}_{1}} \times \frac{\cancel{2}^{1}}{\cancel{3}_{1}} = -1$$

$$= -1$$

∴ The line connecting the midpoints of MN + ML is parallel to NL.

12.



① $M_{AB} = \left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$

$= \left(\frac{-6+1}{2}, \frac{2+3}{2}\right)$

$= \left(\frac{-5}{2}, \frac{5}{2}\right)$

② $m_{CM} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1 - \frac{5}{2}}{-2 + \frac{5}{2}}$

$= \frac{-\frac{2}{2} - \frac{5}{2}}{-\frac{4}{2} + \frac{5}{2}}$

$= \frac{-\frac{7}{2}}{\frac{1}{2}} \rightarrow -\frac{7}{2} \div \frac{1}{2}$

$= -\frac{7}{\cancel{2}} \times \frac{\cancel{2}}{1}$

$= -7$

③ $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{2-3}{-6-1}$

$= \frac{-1}{-7}$

$= \frac{1}{7}$

∴ The slopes of the median + AB are negative reciprocals, so they are perpendicular to each other.

13. $x^2 + y^2 = r^2$ $\quad r = 8$, centre $(0,0)$

$x^2 + y^2 = (8)^2$

$\boxed{x^2 + y^2 = 64}$

14. centre $(0,0)$

$P(12, 5)$

$x^2 + y^2 = r^2$

$(12)^2 + (5)^2 = r^2$

$144 + 25 = r^2$

$\sqrt{169} = \sqrt{r^2}$

$13 = r$

$x^2 + y^2 = r^2$

$x^2 + y^2 = (13)^2$

$\boxed{x^2 + y^2 = 169}$

15. diameter $= 4 \rightarrow$ radius $= 4 \div 2$

$= 2$

centre $(0,0)$

$x^2 + y^2 = r^2$

$x^2 + y^2 = (2)^2$

$\boxed{x^2 + y^2 = 4}$

16. P(3, 4) $x^2 + y^2 = 25 \rightarrow$ radius $= 5$
Q(-4, 3)

$x^2 + y^2 = r^2$
$(3)^2 + (4)^2 = r^2$
$9 + 16 = r^2$
$\sqrt{25} = \sqrt{r^2}$
$5 = r$

$\therefore$ P(3, 4) lies on the circle as the radius is the same length.

$x^2 + y^2 = r^2$
$(-4)^2 + (3)^2 = r^2$
$16 + 9 = r^2$
$\sqrt{25} = \sqrt{r^2}$
$5 = r$

$\therefore$ Q(-4, 3) lies on the circle as the radius is the same length.

17. a) radius increases 15cm/s $t = 4$ s
radius $= (15\text{cm/s})(4\text{ s})$
$= 60$ cm

$x^2 + y^2 = r^2$
$x^2 + y^2 = (60)^2$
$\boxed{x^2 + y^2 = 3600}$

b) $\frac{d}{s} = \frac{\cancel{s}t}{\cancel{s}}$
$t = \frac{d}{s}$
$= \frac{300\text{ cm}}{15\text{ cm/s}}$
$= 20$ seconds

$\therefore$ It will take the ripple 20 seconds to reach the edge of the pool.

c) T(-40, 10)

$x^2 + y^2 = r^2$
$(-40)^2 + (10)^2 = r^2$
$1600 + 100 = r^2$
$\sqrt{1700} = \sqrt{r^2}$
$\sqrt{1700} = r$

$t = \frac{d}{s}$
$= \frac{\sqrt{1700}\text{ cm}}{15\text{ cm/s}}$
$= 2.7$ seconds

$\therefore$ It will take the ripple 2.7 seconds to reach the toy.

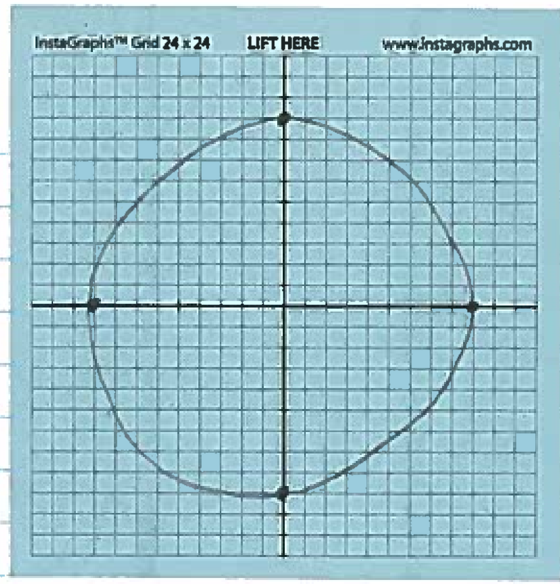
18. $x^2 + y^2 = 81$

radius $= \sqrt{81}$

$= 9$

x-int : $(9,0), (-9,0)$

y-int : $(0,9), (0,-9)$



19. The intersection point of the medians of a triangle is called the centroid.

20. ① Find the perpendicular bisector of each side of the triangle + determine the equation for each. *
② Use two equations + solve using substitution or elimination to determine the intersection point.
③ Verify the intersection point with the third equation.

* To find the perpendicular bisector:
① Find the midpoint of the line segment.
② Determine the slope of the line + find the perpendicul slope.
③ Substitute the perpendicular slope + midpoint into $y = mx + b$ to solve for the y-intercept.
④ Write the equation of the line using the perpendicular slope + the y-intercept $(y = mx + b)$ + then change to standard form.