The Quadratic Formula

The Quadratic Formula is used to find the roots (zeros, x-intercepts) for any quadratic relation *even if it's not factorable*.

If you can factor a quadratic, that's usually faster and easier than using the Quadratic Formula. You can then set $y = 0$ and find the roots.

If you can't factor a quadratic easily, you can convert the quadratic to Standard Form ($y = ax^2 + bx + c$), set $y = 0$, and apply the Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example

Find the roots of $y = x^2 + x - 6$.

Since this is in Standard Form already, we set $y = 0$ and apply the Quadratic Formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1 + 24}}{2}$$

$$x = \frac{-1 \pm \sqrt{25}}{2}$$

$$x = \frac{-1 \pm 5}{2}$$

The roots are $\frac{-1+5}{2} = \frac{4}{2} = 2$ and $\frac{-1-5}{2} = \frac{-6}{2} = -3$.

Example

Find the roots of $y = (x - 1)^2 - 2$.

Since this is **not** in Standard Form, we must first convert it:

$$y = x^2 - 2x + 1 - 2$$

$$y = x^2 - 2x - 1$$

Now apply the Quadratic Formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 4}}{2}$$

$$x = \frac{2 \pm \sqrt{8}}{2}$$

In its simplest form, we can rewrite this as

$$x = 1 \pm \sqrt{2}$$

The roots are $1 + \sqrt{2}$ and $1 - \sqrt{2}$.

Practice

Find the roots of each of the following quadratic relations.

a) $y = x^2 + 7x + 10$

b) $y = 2x^2 - 5x - 3$

c) $y = x^2 - 25$

d) $y = x^2 + 8x + 16$

e) $y = x^2 - 4x - 7$

f) $y = -4x^2 + 6x + 4$

g) $y = -3(x - 1)^2 + 3$

h) $y = 7x^2 - 1 + 6x$