

Name: Solutions

Discrete Probability Distributions Test

MDM4U – Grasley – Fall 2015

Evaluation Summary	
K/A/T	/35
Communication	/5
Overall	/40

1. For each probability distribution, determine the expected value. [4 marks]

k	$P(k)$	$k \cdot P(k)$
0	0.5	0.00
1	0.3	0.30
15	0.15	2.25
100	0.04	4.00
500	0.01	5.00
		<u>11.55</u>

$$E(X) = \sum_{k=1}^n k_i P(k_i)$$

$$= 11.55$$

k	$P(k)$	$k \cdot P(k)$
10	0.2	2
20	0.2	4
30	0.2	6
40	0.2	8
50	0.2	10
		<u>30</u>

$$E(X) = 30.$$

2. Mr. Grasley uses a spreadsheet to generate random numbers between 1 and 999 for a simulation. Explain why the numbers generated have a uniform distribution. [2 marks]

Each number is equally likely to occur (Probability $\frac{1}{999}$), so this is a uniform distribution.

3. Mr. Grasley rolls a special, 10-sided die with the numbers 0 through 9 printed on the faces. Find the expected value for each roll. [3 marks]

$$E(X) = \frac{1}{n} (x_1 + \dots + x_n)$$

$$= \frac{1}{10} (0 + 1 + \dots + 8 + 9)$$

$$= \frac{1}{10} (45)$$

$$= 4.5.$$

The expected value for each roll is 4.5.

4. Explain why the sum of two d6 dice rolls does NOT have a uniform distribution. [2 marks]

Different sums have different probabilities of occurring; for example, $P(2) = \frac{1}{36}$ and $P(3) = \frac{2}{36}$. So this is not a uniform distribution.

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5. The photocopier in the math office has a 99.9% success rate (so on average, 1 in 1000 pages is not successfully printed). If Mr. Grasley prints 80 pages,

a) what is the expected number of failed pages? [2 marks]

$$\begin{aligned} E(X) &= np \\ &= 80(0.001) \\ &= 0.08 \end{aligned}$$

The expected number of failed pages is 0.08.

b) what is the probability that exactly 1 page does not print successfully? [2 marks]

$$\begin{aligned} P(1) &= \binom{80}{1} (0.001)^1 (0.999)^{79} \\ &= 80(0.001)(0.924) \\ &= 0.0739 \end{aligned}$$

There is a 7.39% probability that exactly 1 page does

6. In MDM4U there are 20 students, 8 of whom are left-handed. If Mr. Grasley selects 4 students at random to answer questions on the board, what is the probability that exactly 1 of them is left-handed? [4 marks]

$$\begin{aligned} n &= 20 \\ a &= 8 \\ r &= 4 \\ k &= 1 \end{aligned} \quad P(1) = \frac{\binom{8}{1} \binom{12}{3}}{\binom{20}{4}} = 0.363 \text{ or } 36.3\%$$

The probability that exactly 1 student is left-handed is about 36.3%.

7. Create a probability distribution table for a binomial distribution with $n = 4$ and $p = 0.3$. [5 marks]

k	P(k)
0	0.2401
1	0.4116
2	0.2646
3	0.0756
4	0.0081

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8. A new vaccine is 98% effective. If 200 people are vaccinated,

a) what is the expected number of people who are protected by the vaccine? [2 marks]

$$\begin{aligned} E(X) &= np \\ &= 200(0.98) \\ &= 196 \end{aligned}$$

The expected number of people protected is 196.

b) What is the probability all 200 people are protected by the vaccine? [2 marks]

$$\begin{aligned} P(200) &= \binom{200}{200} (0.98)^{200} (0.02)^0 \\ &= (0.98)^{200} \\ &\approx 0.0176 \end{aligned}$$

The probability that all 200 people are protected is about 1.76%.

9. In the grade 12 class there are 240 students. 215 of those students use a cell phone on a daily basis. If you randomly select a group of 20 students for a survey,

a) what is the expected number of students who use a cell phone on a daily basis? [2 marks]

$$\begin{aligned} n &= 240 \\ a &= 215 \\ r &= 20 \\ E(X) &= r \cdot \frac{a}{n} \\ &= 20 \cdot \frac{215}{240} \\ &\approx 17.917 \end{aligned}$$

About 17.9 students are expected to use a cell phone daily.

b) what is the probability that less than 18 students use a cell phone on a daily basis? [5 marks]

Using the indirect method:

$$\begin{aligned} P(20) &= \frac{\binom{215}{20}}{\binom{240}{20}} \\ &\approx 0.1005 \end{aligned}$$

$$\begin{aligned} P(<18) &= 1 - P(20) - P(19) - P(18) \\ &= 1 - 0.1005 - 0.2563 - 0.2967 \\ &\approx 0.3465 \end{aligned}$$

$$\begin{aligned} P(19) &= \frac{\binom{215}{19} \binom{25}{1}}{\binom{240}{20}} \\ &\approx 0.2563 \end{aligned}$$

The probability that less than 18 students use a cell phone daily is about 34.65%.

$$\begin{aligned} P(18) &= \frac{\binom{215}{18} \binom{25}{2}}{\binom{240}{20}} \\ &\approx 0.2967 \end{aligned}$$

What's Communication?

Use of standard notation and terms

Organization of solutions and ideas

Use of good form and conventions

Completeness and justification of solutions

$$5(c) \quad P(0) = \binom{80}{0} (0.001)^0 (0.999)^{80} \quad [\text{Binomial}]$$

$$= (0.999)^{80} \quad \leftarrow \text{didn't need a fancy formula}$$

$$\approx 0.923$$

$$= 92.3\%$$

The probability that all 80 pages print successfully is about 92.3%.

6(b) This can be calculated in two ways: [Hypergeometric]

$$P(X \geq 2) = P(2) + P(3) + P(4)$$

$$= \frac{\binom{8}{2} \binom{12}{2}}{\binom{20}{4}} + \frac{\binom{8}{3} \binom{12}{1}}{\binom{20}{4}} + \frac{\binom{8}{4} \binom{12}{0}}{\binom{20}{4}}$$

$$= \frac{1848 + 672 + 70}{4845}$$

$$= \frac{2590}{4845}$$

$$= \frac{518}{969}$$

$$\approx 0.53$$

Alternative:

$$P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - [P(0) + P(1)]$$

...

The probability that at least 2 are left-handed is about 53%.