Hypergeometric Probability Distribution

Example problem:

Suppose 30 people have been summoned for jury selection, and that 12 people will be chosen entirely at random (not exactly how the real process works!). Also, suppose that there are 17 male and 13 female candidates. What is the probability that exactly 5 female candidates are chosen?

Solution:

We must choose 5 female candidates from the 13 available, and 7 male candidates from the 17 available. This will be a total of 12 jurors.

Since each person is equally likely to be chosen, the probability that exactly 5 female candidates are chosen is

$$P(5) = \frac{\binom{13}{5}\binom{17}{7}}{\binom{30}{12}}$$
$$\doteq 28.9\%$$

The Distribution

This is an example of the hypergeometric distribution:

- there are n possible outcomes
- there are r trials
- the trials are dependent
- there are a outcomes which are classified as "successes"
- the random variable X measures the number of successes

Then the probability that exactly k successes occur in the r trials is

$$P(X = k) = \frac{\binom{a}{k}\binom{n-a}{r-k}}{\binom{n}{r}}$$

In the jury example above, we have the following parameters:

$n = 30$

$r = 12$

$a = 13$

$k = 5$

Expected Value

$$E(X) = \frac{ra}{n}$$