Example 1: Drawing 2 Face Cards

Suppose you draw 5 cards from a standard, shuffled deck of 52 cards. What is the probability that you draw exactly 2 face cards?

Solution

This is a hypergeometric distribution with the following values:

$n = 52$

$a = 12$

$r = 5$

$k = 2$

$$P(X = k) = \frac{\binom{a}{k}\binom{n-a}{r-k}}{\binom{n}{r}}$$

$$P(X = 2) = \frac{\binom{12}{2}\binom{52-12}{5-2}}{\binom{52}{5}}$$

$$P(X = 2) = \frac{\binom{12}{2}\binom{40}{3}}{\binom{52}{5}}$$

$$P(X = 2) \cong 0.2509$$

The probability of getting exactly two face cards is about 25%.

Example 2: Gender split for hiring

There are 85 people who interview for 4 data science positions at a prestigious company. The company ranks the applicants and decides there are 12 candidates who are all equally suited for the 4 positions. Since they are concerned about unfair biases in the selection process, the company decides to choose 4 people at random from the 12 best candidates.

7 of the 12 candidates identify as female, and 5 of the 12 candidates identify as male. What is the probability that there will be exactly 2 people hired who identify with each gender? What is the expected number hired by gender?

Solution

This is a hypergeometric distribution with the following values:

$n = 12$

$a = 7$

$r = 4$

$k = 2$

Note that we have arbitrarily selected candidates who identify as female as the “success” category; the same procedure works by selecting candidates who identify as male as the “success” category.

$$P(X = k) = \frac{\binom{a}{k}\binom{n-a}{r-k}}{\binom{n}{r}}$$

$$P(X=2)=\frac{\binom{7}{2}\binom{12-7}{4-2}}{\binom{12}{4}}$$
$$P(X=2)=\frac{\binom{7}{2}\binom{5}{2}}{\binom{12}{4}}$$
$$P(X=2)\cong 0.424$$

There is about a 42.4% probability that there will be 2 candidates hired who identify with each gender.

$$\begin{aligned} E(X) &= r\cdot\frac{a}{n} \\ &= 4\cdot\frac{7}{12} \\ &= \frac{7}{3} \\ &\cong 2.33 \end{aligned}$$

The expected number of candidates hired who identify as female is about 2.33, and the expected number of candidates hired who identify as male is about $4-2.33=1.67$.

Example 3: M:tG

Suppose you have a Magic: the Gathering deck of 60 cards, of which 22 are lands and 38 are non-lands. Make a probability distribution table for lands drawn in the opening hand of 7 cards.

Solution

This is a hypergeometric distribution, with the following values (counting land cards as successes):

$n=60$

$a=22$

$r=7$

We need to calculate $P(X=k)$ for each $k\in\{0,1,2,\dots,7\}$.

k	$P(X=k)$
0	3.27%
1	15.73%
2	30.02%
3	29.43%
4	15.98%
5	4.79%
6	0.73%
7	0.04%

For example, the probability of drawing an opening hand with 2 or 3 land cards is $30.02\%+29.43\%\cong 59.46\%$.