# Discrete Binomial Distribution Practice Solutions

1. A new anti-viral medication is known to be 78% effective in curing a particular infection. 18 people are infected and receive the medication.
   a) **How many people are expected** to be cured of the infection by taking the medication?

This is a binomial distribution with $n = 18$ and $p = 0.78$.

$$\begin{aligned} E(X) &= np \\ &= (18)(0.78) \\ &= 14.04 \end{aligned}$$

We expect 14.04 people to be cured of the infection.

   b) What is the probability that **exactly 15** people are cured?

$$\begin{aligned} P(X = 15) &= \binom{18}{15}(0.78)^{15}(0.22)^3 \\ &\doteq 0.209 \\ &= 20.9\% \end{aligned}$$

The probability of exactly 15 people being cured is about 20.9%.

   c) What is the probability that **more than 15** are cured?

$$P(X > 15) = P(X = 16) + P(X = 17) + P(X = 18)$$

We can calculate these each separately and add up the result:

$$\begin{aligned} P(X = 16) &= \binom{18}{16}(0.78)^{16}(0.22)^2 \\ &\doteq 0.139 \\ &= 13.9\% \end{aligned}$$

$$\begin{aligned} P(X = 17) &= \binom{18}{17}(0.78)^{17}(0.22)^1 \\ &\doteq 0.058 \\ &= 5.8\% \end{aligned}$$

$$\begin{aligned} P(X = 18) &= \binom{18}{18}(0.78)^{18}(0.22)^0 \\ &\doteq 0.011 \\ &= 1.1\% \end{aligned}$$

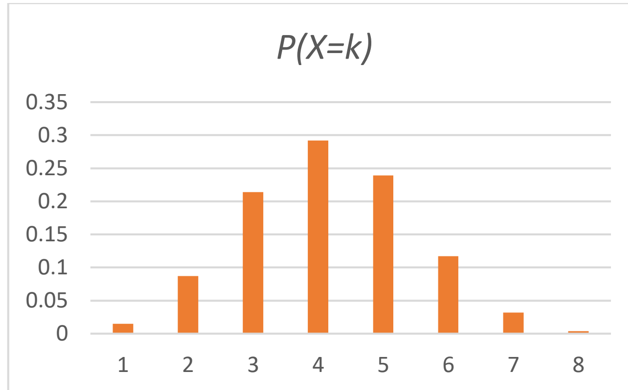
So,

$$\begin{aligned} P(X > 15) &\doteq 13.9\% + 5.8\% + 1.1\% \\ &= 20.8\% \end{aligned}$$

There is about a 20.8% chance that more than 15 people will be cured.

2. Create a **probability distribution table** and a **bar graph** for a binomial distribution with $n = 7$ and $p = 0.45$.

| $k$ | $P(X = k)$ |
|---|---|
| 0 | 0.015 |
| 1 | 0.087 |
| 2 | 0.214 |
| 3 | 0.292 |
| 4 | 0.239 |
| 5 | 0.117 |
| 6 | 0.032 |
| 7 | 0.004 |




3. A manufacturing process has a defect rate of 2%. A quality-assurance technician selects 25 products at random to test.
   a) What is the **expected number** of defective products?

I've decided to have "success" represent the **defective** products (that is, I'm *counting* defective products with the random variable $X$). You could switch it so that you're counting good products instead.

$$\begin{aligned} E(X) &= np \\ &= (25)(0.02) \\ &= 0.5 \end{aligned}$$

We expect about 0.5 defective products.

   b) What is the **probability** that **none** of the products selected will be defective?

$$\begin{aligned} P(X = 0) &= \binom{25}{0}(0.02)^0(0.98)^{25} \\ &\doteq 0.603 \\ &= 60.3\% \end{aligned}$$

The probability of having zero defective products is about 60.3%.

   c) What is the **probability** that **1 or 2** of the products selected will be defective?

We can add $P(X = 1) + P(X = 2)$:

$$\begin{aligned} P(X = 1) &= \binom{25}{1}(0.02)^1(0.98)^{24} \\ &\doteq 0.308 \\ &= 30.8\% \\ P(X = 2) &= \binom{25}{2}(0.02)^2(0.98)^{23} \\ &\doteq 0.075 \\ &= 7.5\% \end{aligned}$$

$$P(X = 1 \text{ or } X = 2) \doteq 30.8\% + 7.5\%$$

$$= 38.3\%$$

The probability of having zero defective products is about 38.3%.

4. The cafeteria has a limited-time promotion. When you purchase the Daily Special you roll 2d6 dice at the checkout. **If you roll two sixes** your Daily Special is free.
   a) If you **buy one Daily Special**, what is the **probability** that it is free?

The probability of rolling two sixes on a single roll is $\frac{1}{36}$. This is the probability that your meal is free.

   b) If you **buy a Daily Special each day** (Monday to Friday), **how many** would you expect to be free?

This is a binomial distribution with $p = \frac{1}{36}$. For part (b), $n = 5$.

$$\begin{aligned} E(X) &= np \\ &= (5)\left(\frac{1}{36}\right) \\ &= \frac{5}{36} \end{aligned}$$

You expect $\frac{5}{36} \doteq 0.14$ of your meals to be free.

   c) A Daily Special is \$5.00. **What do you expect to pay**, on average, if you buy a week's worth of Daily Specials under this promotion?

You expect $\frac{5}{36}$ of a meal to be free, so you expect to pay for $4\frac{31}{36}$ at \$5 each:

$$\left(4\frac{31}{36}\right)(5) \doteq 24.31$$

You expect to pay about \$24.31 on average for a week's worth of specials.

   d) What is the **probability** that you get **more than one** free Daily Special in a **week**?

$$P(X > 1) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$$

We save two probability calculations by using an indirect approach.

$$\begin{aligned} P(X > 1) &= 1 - P(X = 0) - P(X = 1) \\ &= 1 - \binom{5}{0}\left(\frac{1}{36}\right)^0\left(\frac{35}{36}\right)^5 - \binom{5}{1}\left(\frac{1}{36}\right)^1\left(\frac{35}{36}\right)^4 \\ &= 1 - 0.869 - 0.124 \\ &= 0.007 \\ &= 0.7\% \end{aligned}$$

There is about a 0.7% chance of getting more than one free Daily Special in a week.

   e) What is the **probability** that you get **more than one** free Daily Special in a **month** (out of 20 Daily Specials)?

Now the indirect approach is pretty essential, since $n = 20$:

$$
\begin{aligned}
P(X > 1) &= 1 - P(X = 0) - P(X = 1) \\
&= 1 - \binom{20}{0}\left(\frac{1}{36}\right)^{0}\left(\frac{35}{36}\right)^{20} - \binom{20}{1}\left(\frac{1}{36}\right)^{1}\left(\frac{35}{36}\right)^{19} \\
&= 1 - 0.569 - 0.325 \\
&= 0.106 \\
&= 10.6\%
\end{aligned}
$$

There is about a 10.6% chance of getting more than one Daily Special for free in a month.

5. In a half-time contest, one fan gets to shoot free-throws **until they miss one**, then they win a prize **if they hit at least 5 shots**. They have a **75% chance** of hitting each free throw. Explain why this is **NOT** a binomial distribution.

This is not a binomial distribution because there isn't a fixed number of trials ($n$).