Uniform and Binomial Distribution Practice

*For each situation below, determine whether the distribution is **Uniform**, **Binomial**, or **Neither**. If it's Uniform or Binomial, list the parameters (n for Uniform, n and p for Binomial). If there is a question, answer it.*

1. You draw a card from a standard deck of 52 cards. You count 1 point for a red card and 2 points for a black card. What is the expected value of a trial?

This is a uniform distribution – there are two values ($n = 2$), and each has a 50% probability of occurring.

The expected value is

$$\begin{aligned} E(X) &= \frac{1}{2}(1+2) \\ &= \frac{3}{2} \end{aligned}$$

2. You read that 60% of people will not stop to pick up a loonie off the sidewalk. If 40 people each walk past a loonie on the sidewalk, how many would you expect to pick it up? What is the probability that more than 35 people would pick it up?

This is a binomial distribution. "Success" on a trial can be that the person picks up the loonie.

$$n = 40$$

$$p = 0.60$$

$$\begin{aligned} E(X) &= (40)(0.60) \\ &= 24 \end{aligned}$$

We expect 24 people to pick up the loonie.

$$P(X > 35) = P(X = 36) + P(X = 37) + P(X = 38) + P(X = 39) + P(X = 40)$$

There isn't a faster way to make this calculation, since the indirect approach would be a lot worse.

$$\begin{aligned} P(X > 35) &\doteq 0.000024131 + 0.000003913 + 0.000000463 + 0.000000036 + 0.000000001 \\ &= 0.000028544 \end{aligned}$$

This is a very small probability, since it's so far away from the expected value.

3. Grocery stores estimate that 50% of customers who unknowingly purchase spoiled food will return it to the store for an exchange or a refund. If 10 people unknowingly purchase spoiled food, what is the probability that more than half of them do not return it to the store?

This is a binomial distribution with $n = 10$ and $p = 0.5$.

$$\begin{aligned} P(X > 5) &= P(X = 6) + P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10) \\ &\doteq 0.205 + 0.117 + 0.044 + 0.010 + 0.001 \\ &= 0.377 \end{aligned}$$

4. Newly-planted trees have a 93% probability of surviving their first year. If a hedge is planted with 8 trees, what is the probability that all of the trees survive their first year?

Binomial again, with $n = 8$ and $p = 0.93$.

$$\begin{aligned} P(X = 8) &= \binom{8}{8}(0.93)^8(0.07)^0 \\ &\doteq 0.560 \\ &= 56.0\% \end{aligned}$$

5. You flip a coin 5 times. What is the probability that you get more heads than tails?

Binomial with $n = 5$ and $p = 0.5$. Let's call "success" on a trial getting heads.

$$\begin{aligned} P(X > 2) &= 1 - P(X = 0) - P(X = 1) \\ &= 1 - \binom{5}{0}(0.5)^0(0.5)^5 - \binom{5}{1}(0.5)^1(0.5)^4 \\ &= 1 - \frac{1}{32} - \frac{5}{32} \\ &= \frac{26}{32} \\ &= \frac{13}{16} \\ &= 0.8125 \\ &= 81.25\% \end{aligned}$$

6. At a joke shop you buy an **unfair** coin which has a 58% probability of landing on heads and a 42% probability of landing on tails. If you flip it 5 times, what is the probability that you get more heads than tails?

Binomial with $n = 5$ and $p = 0.58$.

$$\begin{aligned} P(X > 2) &= 1 - P(X = 0) - P(X = 1) \\ &= 1 - \binom{5}{0}(0.58)^0(0.42)^5 - \binom{5}{1}(0.58)^1(0.42)^4 \\ &\doteq 1 - 0.013 - 0.090 \\ &= 0.897 \\ &= 89.7\% \end{aligned}$$

7. A casino is considering running a new game. It costs the player \$1.00 to play a round. If the player wins the round, the House (the casino) pays \$2.00 to the player. If the player loses, they win nothing. If the probability of the House winning (and the player losing) is 54%, **what is the expected value** of a round **from the casino's perspective**? What is the probability that the casino **loses money overall** if 5 rounds are played? Is the game **reasonable** for the casino to run?

Binomial distribution with $p = 0.54$.

The House gets \$1.00 per success and loses \$1.00 per failure.

The expected number of successes in a single round ($n = 1$) is

$$\begin{aligned} E(X) &= (1)(0.54) \\ &= 0.54 \end{aligned}$$

The expected value in dollars is

$$\$1.00 \times 0.54 - \$1.00 \times 0.46 = \$0.08$$

If the casino runs 5 rounds ($n = 5$), they have the following profits:

Number of House Wins	**Profit (Loss)**	**Probability**
0	(\$5.00)	0.0206
1	(\$3.00)	0.1209
2	(\$1.00)	0.2838
3	\$1.00	0.3332
4	\$3.00	0.1956
5	\$5.00	0.0459

$$\begin{aligned} P(\text{losing money}) &= P(0) + P(1) + P(2) \\ &= 0.0206 + 0.1209 + 0.2838 \\ &= 0.4253 \\ &= 42.53\% \end{aligned}$$

Deciding whether the casino *should* run this game is fairly complicated. If the expected value of a round were negative, or if the probability of losing money is at least 50%, they should not run the game (or they should change the payouts). As it is, consider what the probability of losing money is on many more plays (say, 1000). We can calculate this using other software. The probability of the casino losing money on 1000 rounds is only about 6%. Casinos operate by mitigating this risk over many, many plays. This game may be reasonable.

Before you ask: no, I would not expect an answer like this to a test question. If the expected profit on a round had been negative you should be able to interpret that as a bad investment for the casino, but I don't let you head onto the InterWebs to find binomial calculators for very large numbers.

Neither uniform nor binomial (nothing special)

k	$P(X = k)$
0	$\frac{2}{5}$
1	$\frac{1}{4}$
2	$\frac{1}{5}$
3	$\frac{1}{10}$
4	$\frac{1}{20}$

Uniform (each probability is equal)

k	$P(X = k)$
-3	$\frac{1}{7}$
-2	$\frac{1}{7}$
-1	$\frac{1}{7}$
0	$\frac{1}{7}$
1	$\frac{1}{7}$
2	$\frac{1}{7}$
3	$\frac{1}{7}$