Confidence Interval Practice

1. A sample of 43 individuals has a mean of $\bar{x} = 23.6$ and a standard deviation of 3.12. Find the 99% confidence interval for the population mean μ.

For a 99% confidence interval, $\alpha = 0.01$ and $1 - \frac{\alpha}{2} = 0.995$.

$$z_{0.995} \doteq 2.58$$

The confidence interval is

$$x \pm \frac{(z_{1-\frac{\alpha}{2}})(s)}{\sqrt{n}}$$

$$\doteq 23.6 \pm \frac{(2.58)(3.12)}{\sqrt{43}}$$

$$\doteq 23.6 \pm 1.23$$

This can also be written as

$$[22.37, 24.83]$$

or

$$22.37 < \mu < 24.83$$

2. A sample has a mean of $\bar{x} = 50$ and a standard deviation of $s = 4$. **[Answers only here; solutions required]**

   a) Find the 95% confidence interval if there are 50 individuals in the sample.

   $$50 \pm 1.11$$

   b) Find the 99% confidence interval if there are 50 individuals in the sample.

   $$50 \pm 1.46$$

   c) Find the 95% confidence interval if there are 300 individuals in the sample.

   $$50 \pm 0.45$$

   d) Find the 99% confidence interval if there are 300 individuals in the sample.

   $$50 \pm 0.60$$

3. A researcher wants to have a margin of error of **at most** 0.10 units for a 95% confidence interval. From prior research they know the standard deviation is 0.34 units. What is the minimum number of individuals needed in the sample to ensure the margin of error does not exceed 0.10?

The margin of error is $\frac{(z_{1-\frac{\alpha}{2}})(s)}{\sqrt{n}}$

To ensure it's below 0.10 we solve

$$\frac{(z_{1-\frac{\alpha}{2}})(s)}{\sqrt{n}} < 0.10$$

$$\frac{(1.96)(0.34)}{\sqrt{n}} < 0.10$$

$$\frac{0.6664}{0.10} < \sqrt{n}$$

$$6.664 < \sqrt{n}$$

$$(6.664)^2 < n$$

$$44.4 < n$$

So n must be at least 45 individuals.

4. A high-performance supplement claims to contain 25g of protein per 30g of product (i.e. $\frac{5}{6}$ of the product is protein). To support the claim, the manufacturer wants to construct a 90% confidence interval. After measuring 40 samples, the mean protein content was 25.21g and the standard deviation was 4.21g. What is the 90% confidence interval?

$$25.21 \pm \frac{(1.96)(4.21)}{\sqrt{40}} \text{ g}$$

$$= 25.21 \pm 1.30 \text{ g}$$