Normal Distribution Practice 2 – Answers

1. Three companies sell vitamin B12 supplements locally. All three products are labelled as containing 100µg (micrograms). **You analyze 60 tablets** from each product and determine the following:

   Product A

   Product B

   Product C

   $\bar{x} = 102.1\text{µg}$

   $\bar{x} = 100.1\text{µg}$

   $\bar{x} = 103.0\text{µg}$

   $s = 1.7\text{µg}$

   $s = 2.4\text{µg}$

   $s = 0.7\text{µg}$

   You want to ensure that **90% of tablets** have between **98µg and 104µg** of vitamin B12. Which products are in the **acceptable range**?

   > For each product we calculate $P(98 < X < 104)$.
   >
   > For Product A, $P(98 < X < 104) \doteq 86\%$.
   >
   > For Product B, $P(98 < X < 104) \doteq 76\%$.
   >
   > For Product C, $P(98 < X < 104) \doteq 92\%$.
   >
   > Only Product C has sufficient tablets in the acceptable range.

2. For a normal distribution with $\bar{x} = 0.243$ and $s = 0.0073$, find $P(0.240 < X < 0.250)$.
   $$P(0.240 < X < 0.250) \doteq 49\%$$

3. A normal distribution has a standard deviation of 3 and a mean of 1. What is the z-score for the value $x = -1$?
   $$z = \frac{-1 - 1}{3}$$
   $$z \doteq - 0.67$$

4. Two confidence intervals are created from the same data. One has a significance level of 10% and the other has a significance level of 5%. Which interval is larger? Give justification.

   > A significance level of 10% results in a smaller critical value (and a smaller margin of error) than a significance level of 5%. This means that the interval for a significance level of 10% is also smaller than that for a significance level of 5%.

5. A normal distribution has a mean of $\mu = 56.3$ and a standard deviation of $\sigma = 12.3$. What value is at the third quartile?

   > The third quartile is a percentile of 75. This means we need to find x so that $P(X < x) = 0.75$.
   >
   > $$z_{0.75} \doteq 0.67$$

$$z_{0.75} = \frac{x - \mu}{\sigma}$$

$$0.67 \doteq \frac{x - 56.3}{12.3}$$

$$x \doteq 64.541$$

6. A government agency is considering changing the nutritional information labels found on food products. Instead of printing a single value for each nutrient, they propose printing a **95% confidence interval** based on the testing of **10 samples** of each product.

a) For a 50g serving of multigrain tortilla chips, the current label shows 13g of total fat. Analysis of 10 samples shows a mean of 12.75g and a standard deviation of 1.22g of fat. Construct the 95% confidence interval for total fat.

       The confidence interval is

$$12.75 \pm 0.76 \text{ g}$$

b) For a 50g serving of multigrain tortilla chips the dietary fibre measured across 10 samples was 1.53g with a standard deviation of 2.75g. Construct the 95% confidence interval for dietary fibre and explain why the interval is not valid.

       The confidence interval is

$$1.53 \pm 1.70 \text{ g}$$

       This can be rewritten as

$$[-0.17 \text{ g}, 3.23 \text{ g}]$$

       Since a value for dietary fibre in a food can't be negative, this confidence interval is not valid. Likely the distribution of values is not a normal distribution and can't be interpreted this way.

c) Propose some reasons that this labelling change may not be beneficial for the consumer. Give at least two reasons and include an explanation for each.

       Examples of reasons: increased confusion about food contents, wide/invalid ranges (see part b), consumers may use minimum/maximum values inappropriately (lowest fat, highest fibre), etc.