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Normal Distribution Review Solutions

1. A survey of 850 high school students asked how many hours per day they spent using social media services. The mean was $\bar{x} = 3.67$ hours and the standard deviation was $s = 1.19$ hours.
   a) What **percentage** of surveyed students used social media services between 3 and 4 hours per day?

For 3 hours,

$$z = \frac{3 - 3.67}{1.19}$$

$$z \doteq -0.56$$

$$P(X < 3) \doteq 0.2877$$

For 4 hours,

$$z = \frac{4 - 3.67}{1.19}$$

$$z \doteq 0.28$$

$$P(X < 4) \doteq 0.6103$$

Therefore

$$P(3 < X < 4) \doteq 0.6103 - 0.2877$$

$$= 0.3226$$

$$= 32.26\%$$

Approximately 32.26% of surveyed students use social media services between 3 and 4 hours per day.

   b) **How many hours** per day did students in the 75th percentile use social media services?

$$z_{0.75} \doteq 0.67$$

$$z_{0.75} = \frac{x - \bar{x}}{s}$$

$$0.67 \doteq \frac{x - 3.67}{1.19}$$

$$x \doteq 4.4673$$

Students in the 75th percentile used social media services about 4.5 hours per day.

   c) **How many of the 850 students** surveyed use social media services for less than 2 hours per day?

$$z = \frac{2 - 3.67}{1.19}$$

$$z \doteq -1.40$$

$$P(X < 2) \doteq 0.0808$$

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8.08% of 850 students is 68.68, or approximately 69 students.

2. For each normal distribution, find the indicated value. **[only answers shown – solutions required]**
   a) $\mu = 20, \sigma = 3$, find $P(X < 21)$.

$$P(X < 21) = 0.6293$$

   b) $\mu = 0.290, \sigma = 0.087$, find $P(0.200 < X < 0.350)$.

$$P(0.200 < X < 0.350) = .6034$$

   c) $\bar{x} = 121, s = 10$, find $P(X < 115 \text{ or } X > 119)$.

$$P(X < 115 \text{ or } X > 119) = 0.8536$$

   d) $u = 50, P(X > 45) = 92\%$, find σ.

$$\sigma = 3.55$$

   e) $\mu = 75, \sigma = 5$, find x so that $P(X < x) = 19\%$.

$$x = 70.6$$

3. For a normal distribution with mean 100 and standard deviation 8, what percentile is the value 102?

$$z = \frac{102 - 100}{8}$$

$$z = 0.25$$

$$P(X < 102) = 0.5987$$

The value 102 is at about the 60th percentile.

4. A study of a sample of teen elite athletes showed elevated levels of creatine kinase (CK) after two days' rest.

   The mean for the sample was $\bar{x} = 550$ U/L, which was outside the reference range of 94 to 499 U/L.

   There were 17 participants in the study, and the standard deviation for CK was 73 U/L.

   Construct the 95% confidence interval for the mean level of CK in the teen elite athlete population.

$$\alpha = 0.05$$

$$n = 17$$

$$\bar{x} = 550$$

$$s = 73$$

The confidence interval is

$$\bar{x} \pm z_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

The z-score is

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$$z_{1-\frac{\alpha}{2}} = z_{0.975}$$
$$= 1.96$$

Therefore the confidence interval is

$$550 \pm \frac{(1.96)(73)}{\sqrt{17}}$$

$$550 \pm 34.7 \text{ U/L}$$

5. Construct the confidence interval for the population mean using a sample mean of $\bar{x} = 26.32$, a standard deviation of $s = 4.13$, and a sample size of $n = 36$ if
   a) $\alpha = 0.01$

$$z_{0.995} = 2.57$$

The confidence interval is

$$26.32 \pm \frac{(2.57)(4.13)}{\sqrt{36}}$$

$$26.32 \pm 1.77$$

   b) $\alpha = 0.05$

$$z_{0.975} = 1.96$$

The confidence interval is

$$26.32 \pm \frac{(1.96)(4.13)}{\sqrt{36}}$$

$$26.32 \pm 1.35$$

6. A pilot study shows a mean of 187.81 units and a standard deviation of 3.49 units. In a follow up study, researchers want to have a margin of error that is no more than 1.00 units for a 95% confidence interval. What is the minimum number of participants they should have in the follow up study that will achieve this goal if the standard deviation does not change?

The margin of error is

$$z_{0.975}\frac{s}{\sqrt{n}} = \frac{(1.96)(3.49)}{\sqrt{n}}$$
$$= \frac{6.8404}{\sqrt{n}}$$

We want this to be less than 1, so

$$\frac{6.8404}{\sqrt{n}} < 1.00$$

$$6.8404 < \sqrt{n}$$

Since $\sqrt{n}$ is positive. Then

$$(6.8404)^2 < n$$

$$46.79 \dot{<} n$$

So the number of participants must be at least 47.